THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010J/K University Mathematics 2016-2017
Assignment 1 (Due date: 28 Sept, 2017)

1. Let A, B and C be three sets. Show that $A \backslash (B \cup C) = (A \backslash B) \backslash C$.

   Give an example of sets A, B and C to show that $A \backslash (B \backslash C)$ and $(A \backslash B) \backslash C$ are not the same.

2. Let m and n be two integers. Suppose that A and B are the sets of integers which are divisible by m and n.

   Find $A \cap B$ and prove your assertion.

   (Hint: To prove two sets are equal, you may show each of them is a subset of the other one.)

3. Prove that the sum of a rational number and an irrational number is irrational.

4. Let n be integer. Prove that if n^2 is odd, then n is odd.

5. Define a relation $\sim$ on $\mathbb{R}^2$ such that $(x_1, y_1) \sim (x_2, y_2)$ if and only if $y_2 - y_1 = x_2 - x_1$.

   (a) Show that the relation $\sim$ is an equivalence relation.

   (b) What are the elements of the equivalence class $[(0, 0)]$?

6. Let $\mathbb{R}[x]$ be the set of all polynomials with real coefficients.

   Define a relation $\sim$ on $\mathbb{R}[x]$ such that $P(x) \sim Q(x)$ if and only if $P(x) - Q(x)$ is divisible by $x - 1$.

   (a) Show that the relation $\sim$ is an equivalence relation.

   (b) What are the elements of the equivalence class $[2]$?

7. Define an equivalence relation $\sim$ on $\mathbb{N}^2$ defined by $(m, n) \sim (p, q)$ if and only if $m + q = p + n$ and define $\mathbb{Z} = \mathbb{N}^2 / \sim$.

   A multiplication $*$ on $\mathbb{N}^2$ is defined by $(m, n) * (p, q) = (m \cdot p + n \cdot q, n \cdot p + m \cdot q)$, where $+$ and $\cdot$ are addition and multiplication defined on $\mathbb{N}$.

   (a) Show that the multiplication $*$ on $\mathbb{N}^2$ induces a multiplication on $\mathbb{Z}$.

   (b) Show that the induced multiplication is commutative and associative (by assuming every properties of $\mathbb{N}$).

8. (Optional) Let C^∞ be the set of all functions from $\mathbb{R}$ to $\mathbb{R}$ that are differentiable for any number of times. Suppose that $a \in \mathbb{R}$ and define a relation on $\sim$ on C^∞ such that $f \sim g$ if and only if $f(a) = g(a)$ and $f'(a) = g'(a)$.

   Show that the relation $\sim$ is an equivalence relation.

9. (Optional) Let $P(x)$ be a statement function of x. The statement 'There exists unique x such that P(x)' (denoted by '$\exists! x, P(x)$') can be expressed as '$\exists x, P(x) \wedge \big(\forall y, P(y) \to (y = x)\big)$'.

   Write down the negation of the above statement in symbolic and verbal way.